NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	**LEVEL:** 6
COURSE CODE: SIN601S	**COURSE NAME:** STATISTICAL INFERENCE 2
SESSION: JUNE 2019	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr D.B. GEMECHU
MODERATOR:	Dr C.R. KIKAWA

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

QUESTION 1 [20 marks]

1. Let $Y_1 < Y_2 < \cdots < Y_{11}$ be the order statistics of 11 independently and identically distributed continuous random variables $X_1, X_2, \ldots, X_{11}$ with pdf f given by

$$f_X(x) = \begin{cases} \frac{1}{2} & for\ 0 < x < 2 \\ 0 & otherwise \end{cases}$$

Then find

1.1. The pdf of the r^{th} order statistics. **Hint:** $f_{Y_r}(y) = nf_X(y)\binom{n-1}{r-1}[F_X(y)]^{r-1}[1 - F_X(y)]^{n-r}$ **[3]**

1.2. The pdf of the minimum order statistics **[3]**

1.3. The pdf of the maximum order statistics **[3]**

1.4. The pdf of the median **[3]**

1.5. The joint pdf of $Y_1, Y_2, \ldots, Y_{11}$ **[3]**

1.6. If the number of random variables are reduced to 3, thus, X_1, X_2, X_3, then find the joint pdf of the minimum and maximum order statistics. **[5]**

Hint: $f_{Y_i, Y_j}(y_i, y_j) = \frac{n!}{(i-1)!(j-i-1)!(n-j)!}[F_X(y_i)]^{i-1}f_X(y_i)[F_X(y_j) - F_X(y_i)]^{j-i-1}f_X(y_j)[1 - F_X(y_j)]^{n-j}$

QUESTION 2 [8 marks]

2. Suppose a random variable Z has a standard normal distribution with mean 0 and variance 1 and a random variable Y has a normal distribution with mean 1 and variance 1. If the random variables Z and Y are assumed to be independent, then what is the distribution of $\frac{Z+Y}{2}$? (**Hint:** If $X \sim N(\mu, \sigma^2)$, then $\mathrm{M_X(t)} = \mathrm{e}^{\mu t + \frac{\sigma^2 t^2}{2}}$)) **[8]**

QUESTION 3 [20 marks]

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Gamma distribution with parameters α and θ, that is

$$f(x_i|\alpha, \theta) = \begin{cases} \frac{1}{\Gamma(\alpha)\theta^\alpha} x_i^{\alpha-1} e^{-\frac{x_i}{\theta}} & for\ x_i > 0; \alpha > 0;\ \theta > 0 \\ 0 & otherwise \end{cases}$$

If α is known, then

3.1. Show that the moment generating function of X_i is given by $M_{X_i}(t) = \left(\frac{1}{1-\theta t}\right)^\alpha$ **[6]**

3.2. Find the methods of moments estimator of θ. **[6]**

3.3. Find the maximum likelihood estimator of θ **[6]**

3.4. What is the maximum likelihood estimator of $\alpha\theta^2$? **[2]**

QUESTION 4 [6 marks]

4. Observations $Y_1, \ldots, Y_n$ are assumed to come from a model with $E(Y_i) = 2 + \beta x_i^2$, where β is an unknown parameter and $x_1, x_2, \ldots, x_n$ are given constants. Then find the least square estimator of the parameter β. **[6]**

QUESTION 5 [16 marks]

5. Let $X_1, ..X_n$ be a random sample of observations from a population with mean μ and variance σ^2. Consider the following two point estimators of μ

$$\hat{\theta}_1 = \frac{1}{8}X_1 + \frac{3}{4(n-2)}(X_2 + X_3 + \cdots + X_{n-1}) + \frac{1}{8}X_n \qquad and \qquad \hat{\theta}_2 = \bar{X}$$

5.1. Show that both estimators are unbiased estimators of μ. **[6]**

5.2. Find the efficiency of $\hat{\theta}_2$ relative to $\hat{\theta}_1$. **[10]**

QUESTION 6 [15 marks]

6. If $X_1, X_2, . . ., X_n$ be a random sample from the Poisson distribution with the parameter θ, then

6.1. Show that $\bar{X}$ is a minimum variance unbiased estimator (MVUE) of θ. **[10]**

6.2. Show that $\bar{X}$ is also a consistent estimator of θ. **[4]**

Hint: If $X \sim Poisson(\theta)$, then

$$f(x|\theta) = \begin{cases} \dfrac{\theta^x e^{-\theta}}{x!} & for\ x = 0,1,2, \ldots, \infty \\ 0 & otherwise \end{cases} \qquad \text{and } E(X) = Var(X) = \theta$$

QUESTION 7 [15 marks]

7. Suppose that $X_1, . . . , X_n$ are random samples from normal distribution $N(\mu,\ 1)$ with probability density function given by

$$f(x_i|\mu) = \begin{cases} \dfrac{1}{\sqrt{2\pi}} exp\left[-\dfrac{1}{2}(x_i - \mu)^2\right] & for\ -\infty < x_i < \infty \\ 0 & otherwise \end{cases}$$

Furthermore, suppose that μ has a prior distribution with mean 0 and variance 1 with pdf

$$h(\mu) = \begin{cases} \dfrac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\mu^2} & for\ -\infty < \mu < \infty \\ 0 & otherwise \end{cases}$$

If the squared error loss function is used, show that the Bayes' estimator of μ is given by $\frac{\sum_{i=1}^{n} x_i}{n+1}$.

[15]

=== END OF PAPER===

TOTAL MARKS: 100